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Adomian Decomposition Approach to a Mathematical Modeling and Analysis of Potentiometric and Amperometric Enzyme Electrodes

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ABSTRACT

A mathematical model to describe the concentration of profiles and flux for potentiometric and amperometric enzyme electrodes and of enzyme reactors has been developed. This model contains a non-linear term related to Micahlies-Menten kinetics. Analytical expressions pertaining to the substrate concentration and product concentration were reported for all values of parameters γ_S, γ_P and α. In this work, we report the theoretically evaluated steady state current for short and small values of saturation parameter α and reaction-diffusion parameters γ_S, γ_P. This is done by using Adomian decomposition method. These analytical results were found to be in good agreement with numerical results.

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1. INTRODUCTION

A biosensor is an analytical device that inverts a biochemical reaction process into a measurable signal using transducer [1–3]. Biosensors can be used for detecting various substances like pollutants, metabolites, microbial load, etc. Usually, a biosensor consists of two elements: a biological sensing element and a transducer for detecting the analyte concentration. Biosensors have a lot of advantages compared to usual biological methods of analysis - biosensors are small, simple to use, radioactivity proof, etc. These characteristics make them attractive to use [4].

An electrochemical biosensor, when biochemical reactions between an immobilized biomolecule and target analyte produce or consume ions or electrons, which affects the measurable electric current [5]. Electrochemical biosensors are divided into amperometric and potentiometric ones. Amperometric biosensors are most widely used, they are very sensitive and more suitable for mass production than the potentiometric ones [6–10]. The amperometric biosensor is an electronic signal converter with biochemically active substance usually an enzyme. The operation of the amperometric biosensors is based on calculating the Faraday current, which is calculated while the current at the electrode is set constant. The current arises because of the oxidation or reduction of the product [11–13]. Generally, the process is modeled using Michael-Menten kinetic equations.

As biocatalytic reaction rates are often chosen to be first order dependent on the bulk analyte concentration, such steady-state currents are usually proportional to the bulk analyte concentration. Potentiometric measurements involve determination of the potential difference between either an indicator or a reference electrode [14], or two reference electrodes separated by a perm selective membrane when there is no significant current flowing between them. The transducer may be an ion-selective electrode (ISE), which is an electrochemical sensor based on thin films or selective membranes as recognition elements Buck and Lindner, 1994 [15].

The most common potentiometric devices are pH electrodes; the potential differences between these indicator and reference electrodes are proportional to the logarithm of the ion activity or gas fugacity (or concentration), as described by the Nernst-Donnan equation. This is only the case when the membrane or layer selectivity is infinite or if there is a constant or low enough concentration of interfering ions; and potential differences at various phase boundaries are either

negligible or constant, except at the membrane: sample-solution boundary. When a biocatalyst layer is placed adjacent to the potentiometric detector, one has to take into account of, as for any biocatalyst sensor: transport of the substrate to be analyzed to the biosensor surface; analyte diffusion to the reacting layer; analyte reaction in the presence of biocatalyst and diffusion of reaction product towards both the detector and the bulk solution. Morf presented a relatively simple approach for the electrode response that applies to the whole range of substrate concentrations by obtaining an explicit result and also he described the principles theoretical treatment and numerical simulation of potentiometer and amperometric enzyme electrodes and of enzyme reactors [16].

To my knowledge no rigorous analytical solutions for non-linear steady state concentration and flux for potentiometric and amperometric enzyme electrodes for all values of γ_S, γ_P and α have been reported. It should be pointed out that, complete solutions have not yet been obtained even for steady state behavior because of the nonlinearity inherent in Michalies-Menten kinetics. In this paper, we have derived new, simple analytical expressions of concentration and current in order to describe and evaluate the performances of potentiometric and amperometric enzyme electrodes using Adomian decomposition method.

2. Mathematical formulations of analysis and problems

In this system, the substrate molecules diffuse into the membrane phase where they react according to the Michalies- Menten type reaction in Eq. (4) to yield the product *P*:

$$E + S \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES \xrightarrow{k_3} E + \nu P \qquad (4)$$

$$K_M = \frac{k_2 + k_3}{k_1} \qquad (5)$$

Where, *ES* is the intermediate enzyme-substrate complex, k_1, k_2 and k_3 are the rate constants of the respective partial reactions, and K_M is the Michalies constant [17] defined in Eq. (5) [18. Space and time-dependent influences of diffusion and reaction processes in the enzyme

membrane are described by Eqs. (6-9), which are valid for the species, S and P respectively [18]:

$$\frac{\partial [S]_{em}}{\partial t} = D_S \frac{\partial^2 [S]_{em}}{\partial x^2} - k_3 [E]_{tot} \frac{[S]_{em}}{[S]_{em} + K_M} \tag{6}$$

$$\frac{\partial [P]_{em}}{\partial t} = D_P \frac{\partial^2 [P]_{em}}{\partial x^2} + \nu k_3 [E]_{tot} \frac{[S]_{em}}{[S]_{em} + K_M} \tag{7}$$

Where, $[E]_{tot}$ is the total enzyme concentration, $[S]_{em}, [P]_{em}$.are the concentrations of the species in the enzyme membrane, ν is the number of products species obtained per substrate molecule. Eqs. (6) and (7) are solved for the following boundary conditions by assuming the zero fluxes at $x = 0$ and of equilibrium distribution at $x = d$. The boundary conditions are

$$\frac{\partial [S]_{em}}{\partial x}(x = 0) = 0, \qquad \frac{\partial [P]_{em}}{\partial x}(x = 0) = 0 \tag{8}$$

$$[S]_{em}(x = d) = k_s [S]_{aq}, \qquad [P]_{em}(x = d) = k_p [P]_{aq} \tag{9}$$

For enzyme reactors, the outward flux of product species at $x = d$ is described by:

$$J_P = -D_P \left. \frac{\partial [P]_{em}}{\partial x} \right|_{x=d} \tag{10}$$

We introduce the following dimensionless parameter as:

$$S = \frac{[S]_{em}}{k_s [S]_{aq}}, \; P = \frac{[P]_{em}}{k_s [S]_{aq}}, \; \xi = \frac{x}{d}, \quad \alpha = \frac{k_s [S]_{aq}}{K_M}, \; \gamma_S = \frac{kd^2}{D_S}, \quad \gamma_P = \frac{kd^2}{D_P}, \; \kappa = \frac{k_3 [E]_{tot}}{K_M} \tag{11}$$

The above non-linear ordinary differential equations (Eqs. (6-10)) in dimensionless form

$$\frac{d^2 S}{d\xi^2} - \frac{\gamma_E S}{1 + \alpha S} = 0 \tag{12}$$

$$\frac{d^2 P}{d\xi^2} + \frac{\nu \gamma_S S}{1 + \alpha S} = 0 \tag{13}$$

The corresponding boundary conditions are

$$\frac{dS}{d\xi}=0,\quad \frac{dP}{d\xi}=0,\quad \xi=0 \tag{14}$$

$$S=1,\quad P=m,\qquad \xi=1 \tag{15}$$

The dimensionless current is given by

$$J=-\frac{J_P d}{D_P k_S S_{aq}}\left.\frac{dP}{d\xi}\right|_{\xi=1} \tag{16}$$

3. Solution of the boundary value problem using the Adomian decomposition method

In the recent years, much attention is devoted to the application of the Adomian decomposition method to the solution of various scientific models [19]. An efficient modification of the standard adomian decomposition method for solving singular initial value problem in the second order ordinary differential equation. The ADM yields, without linearization, perturbation, transformation or discretization, an analytical solution in terms of a rapidly convergent infinite power series with easily computable terms. The decomposition method is simple and easy to use and produces reliable results with little iteration used. The results show that the rate of convergence of Adomian decomposition method is higher than standard Adomian decomposition method [20-24]. Using this method (see appendix A), we can obtain the analytical expression of concentrations as follows:

$$S(r)=1+\frac{5\gamma_S^2}{24(1+\alpha)^3}-\frac{\gamma_S}{2(1+\alpha)}+\frac{\gamma_S r^2}{2(1+\alpha)}-\frac{\gamma_S^2 r^2}{4(1+\alpha)^3}+\frac{\gamma_S^2 r^4}{24(1+\alpha)^3} \tag{17}$$

$$P(r)=m-\frac{5\nu\gamma_S\gamma_P}{24(1+\alpha)^3}+\frac{\nu\gamma_P}{2(1+\alpha)}-\frac{\nu\gamma_P r^2}{2(1+\alpha)}+\frac{\nu\gamma_S\gamma_P r^2}{4(1+\alpha)^3}-\frac{\nu\gamma_S\gamma_P r^4}{24(1+\alpha)^3} \tag{18}$$

Equations (17-18) are the analytical solutions for the dimensionless concentrations as a function of dimensionless distance r. The current density is given by

$$J = \frac{v\gamma_S\gamma_P}{3(1+\alpha)^3} - \frac{v\gamma_P}{(1+\alpha)} \qquad (19)$$

Equation (19) represents the new approximate analytical expression for the current for short and small values of saturation parameter α and reaction-diffusion parameters γ_S, γ_P.

4. NUMERICAL SIMULATION

The non-linear differential equations (Eqs (12) and (13)) for the boundary conditions (Eqs (14) and (15)) are also solved numerically. We have used the function pdex4 in MATLAB software to solve numerically the initial-boundary value problems for the nonlinear differential equations. This numerical solution is compared with our analytical results in Figs (1) to (5). Upon comparison, it gives a satisfactory agreement for all values of the dimensionless parameters, γ_S, γ_P and α. The MATLAB program is also given in appendix B.

5. RESULTS AND DISCUSSION

The primary result of this work is the first accurate calculation of steady state concentration of substrate (or product) and current for all values of γ_S, γ_P and α for potentiometric response of enzyme electrode system. The concentrations $S(r)$ are represented in Figs. 1*(a)* –(b) and 2*(a)*–(b). From these figures, it is evident that the value of concentration gradually increases as the saturation parameter α increases. The concentration increases as the distance increases and attains the maximum value 1. Figure 3 represent the concentration profile of $P(r)$. It is clear that as reaction-diffusion parameter γ_P increases the value of dimensionless concentration $P(r)$ is also increased. Fig 4 represents the saturation parameter α decreases the value of concentration $P(r)$ is also increased. The analytical expression for dimensionless current is given in Eq. (16). The dimensionless current J versus substrate molecule v is given in Fig. 5. The value of the current increases when substrate molecule v increases and γ_P decreases. In Fig. 6, the value of current increases when saturation parameter α decreases.

6. CONCLUSION

The steady state potentiometric response for an enzyme electrode system which exhibits Michalies-Menten kinetics has been discussed. Approximate analytical solution to the nonlinear reaction-diffusion equation has been presented using Adomian decomposition method. A simple and a new method for estimating the concentration of substrate or product and the corresponding current for all values of $\alpha, \gamma_S, \gamma_P$ and ν has been suggested. The solution procedure can be easily extended to all kinds of non-linear equations with various complex boundary conditions in enzyme-substrate reaction-diffusion processes.

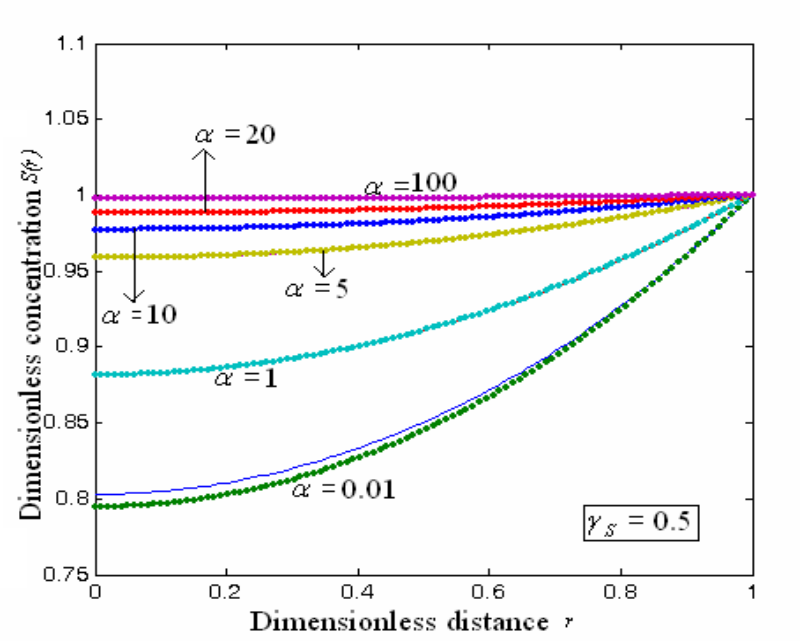

Fig. 1 (a): Normalized concentration profile $S(r)$ as a function of dimensionless parameter r. The concentrations were computed using Eq. (17) for various values of the α and for the fixed values of $\gamma_S = 0.5$, (—) denotes Eq. (17) and (…) denotes the numerical simulation.

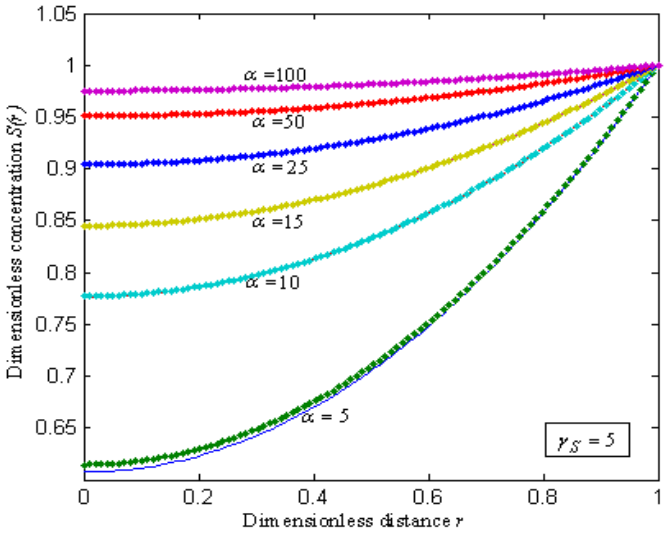

Fig. 1(b): Normalized concentration profile $S(r)$ as a function of dimensionless parameter r. The concentrations were computed using Eq. (17) for various values of the α and for the fixed values of $\gamma_S = 5$, (—) denotes Eq. (17) and (…) denotes the numerical simulation.

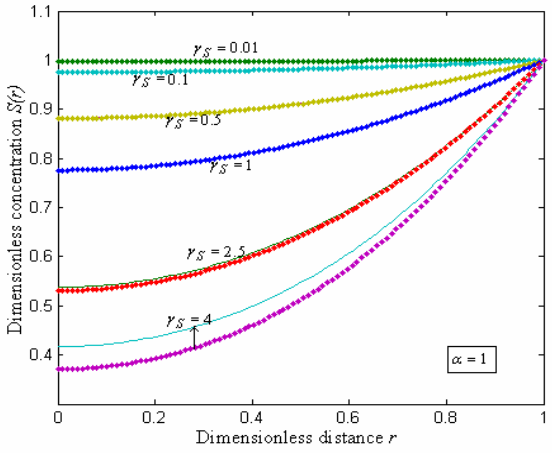

Fig. 2(a): Normalized concentration profile $S(r)$ as a function of dimensionless parameter r. The concentrations are computed using Eq. (17) for various values of the γ_S and for the fixed values of $\alpha = 1$. (—) denotes Eq. (17) and (…) denotes the numerical simulation.

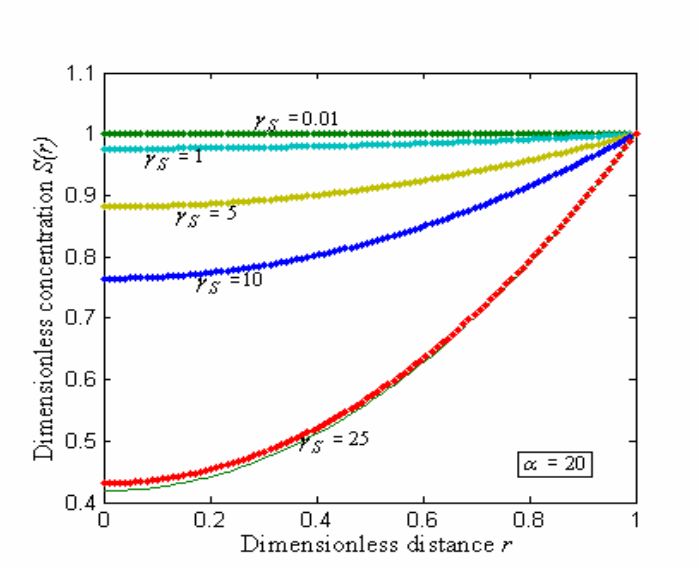

Fig. 2(b): Normalized concentration profile $S(r)$ as a function of dimensionless parameter r. The concentrations are computed using Eq. (17) for various values of the γ_S and for the fixed values of $\alpha = 20$. (—) denotes Eq. (17) and (…) denotes the numerical simulation.

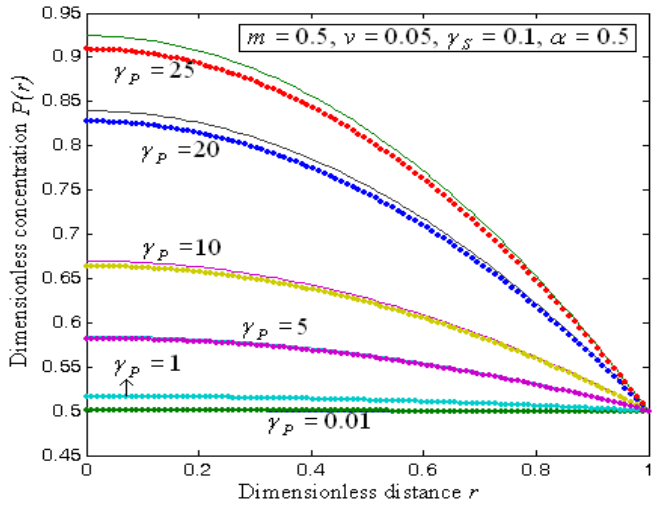

Fig. 3: The dimensionless concentration $P(r)$ versus the dimensionless distance r for various values of γ_P and some fixed value of m, v, γ_S, α. The concentrations were computed using Eq. (18). The key to the graph: (….) represents the Eq. (18) and (—) represents the numerical simulation.

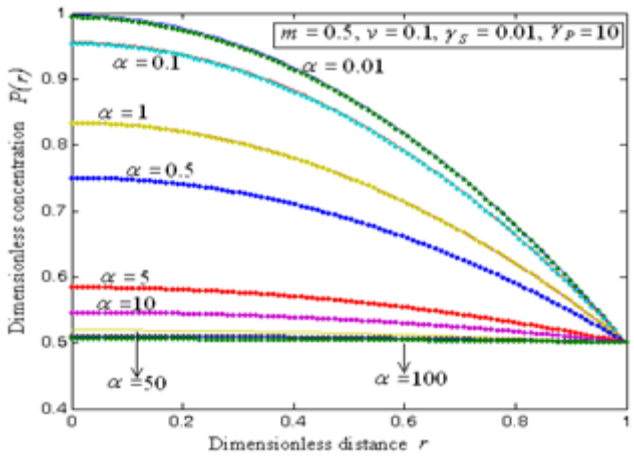

Fig. 4: The dimensionless concentration $P(r)$ versus the normalized distance r for various values of α and some fixed value of $m, \nu, \gamma_S, \gamma_P$. The concentrations were computed using Eq. (18). The key to the graph: (….) represents the Eq. (18) and (—) represents the numerical simulation.

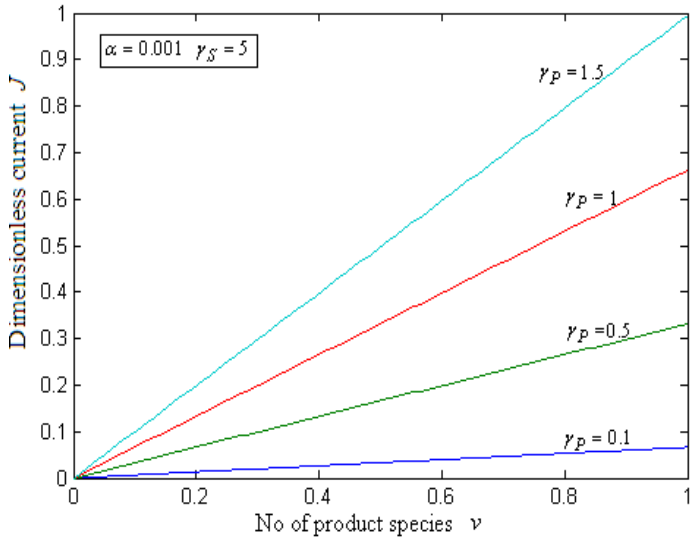

Fig. 5: The normalized current J versus substrate molecule ν using Eq. (19) for various values of reaction-diffusion parameter γ_P and fixed values of $\alpha = 0.001$, $\gamma_S = 5$.

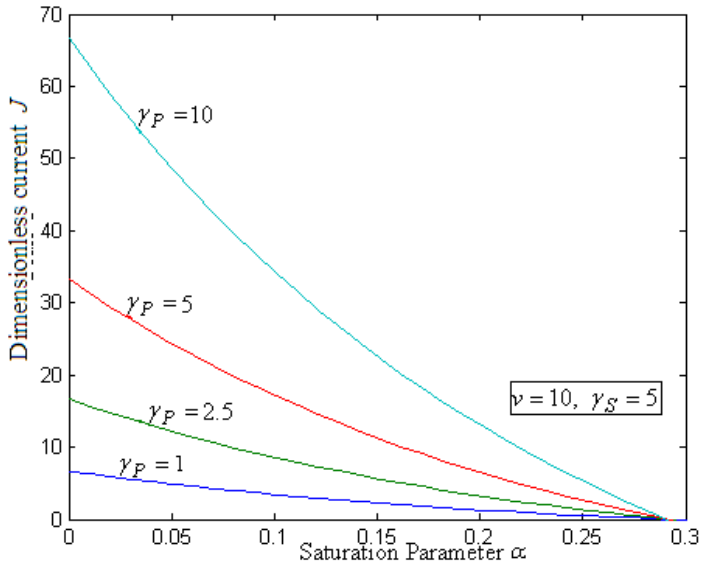

Fig. 6: Variation of normalized current J versus saturation parameter α using Eq. (19) for various values of reaction-diffusion parameter γ_P and fixed values of $\alpha = 10,\ \gamma_S = 5$.

Appendix A

Adomian decomposition method [28-32] depends on the non-linear differential equation

$$F(x, y(x)) = 0 \qquad \text{(A1)}$$

into the two components

$$L(y(x)) + N(y(x)) = 0 \qquad \text{(A2)}$$

Where, L and N are the linear and non-linear parts of F respectively. The operator L is assumed to be an invertible operator. Solving for $L(y)$ leads to

$$L(y) = -N(y) \qquad \text{(A3)}$$

Applying the inverse operator L on both sides of Eq. (A3) yields

$$y = -L^{-1}(N(y)) + \varphi(x), \qquad \text{(A4)}$$

Where, $\varphi(x)$ is the constant of integration which satisfies the condition $L(\varphi) = 0$. Now assuming that the solution y can be represented as infinite series of the form

$$y = \sum_{n=0}^{\infty} y_n \qquad \text{(A5)}$$

Furthermore, suppose that the non-linear term $N(y)$ can be written as infinite series in terms of the Adomian polynomials A_n of the form

$$N(y) = \sum_{n=0}^{\infty} A_n \qquad \text{(A6)}$$

Where, the Adomian polynomials A_n of $N(y)$ are evaluated using the formula:

$$A_n(x) = \frac{1}{n!}\frac{d^n}{d\lambda^n} N\left(\sum_{n=0}^{\infty} \lambda^n y_n\right)\Big|_{\lambda=0} \qquad \text{(A7)}$$

Then substituting Eqns. (A5) and (A6) in Eq. (A4) gives

$$\sum_{n=0}^{\infty} y_n = \varphi(x) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \qquad \text{(A8)}$$

Then equating the terms in the linear system of Eq. (A8) gives the recurrent relation

$$y_0 = \varphi(x) y_{n+1} = -L^{-1}(A_n) \qquad n \geq 0 \qquad \text{(A9)}$$

However, in practice all the terms of series in Eq. (A7) cannot be determined, and the solution is approximated by the truncated series $\sum_{n=0}^{\infty} y_n$. This method has been proven to be very efficient in solving various types of non-linear boundary and initial value problems.

Appendix B

Scilab/ Matlab program to find the numerical solution of Eqs. (12-15):

function pdex4

```
m = 0;
x = linspace(0,1);
t=linspace(0,1000);
sol = pdepe(m,@pdex4pde,@pdex4ic,@pdex4bc,x,t);
u1 = sol(:,:,1);
u2 = sol(:,:,2);
figure
plot(x,u1(end,:))
title('u1(x,t)')
xlabel('Distance x')
ylabel('u1(x,2)')
plot(x,u2(end,:))
title('u2(x,t)')
xlabel('Distance x')
ylabel('u2(x,2)')
function [c,f,s] = pdex4pde(x,t,u,DuDx)
c = [1; 1];
f = [1; 1] .* DuDx;
alpha=5;
y = (u(1)/(1+alpha*u(1)));
gamma=100;
gamma1=1;
v=1.5;
F =(-gamma*y);
F1 =(v*gamma1*y);
s=[F;F1];
function u0 = pdex4ic(x);
u0 = [1; 0];
function[pl,ql,pr,qr]=pdex4bc(xl,ul,xr,ur,t)
pl = [0; 0];
ql = [1; 1];
```

```
pr = [ur(1)-1; ur(2)-0.5];
qr = [0; 0];
```

Appendix C

Nomenclature and units

Symbol	Mean ing	Usual dimension
v	No of product species	none
D_S	Diffusion Coefficient of the substrate	$cm^2\ sec^{-1}$
D_P	Diffusion Coefficient of the product	$cm^2\ sec^{-1}$
K_M	Michalies constant	mole cm^{-3}
k_3	Rate constant for irreversible step	sec^{-1}
α	Saturation parameter	none
γ_S , γ_P	Reaction-diffusion parameters	none
$[S]_{em}$	Concentration of the substrate	mole cm^{-3}
$[P]_{em}$	Concentration of the product	mole cm^{-3}
$[S]_{aq}$	Concentration of substrate in the sample	mole cm^{-3}
$[P]_{aq}$	Concentration of substrate in the product	mole cm^{-3}
x	Distance	cm
r	Dimensionless distance	none
S, P	Dimensionless substrate concentrations	none
m	Dimensionless substrate concentration at particle surface	none

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